Practice 9, 10

Topic: ***Check of ACS on stability by the second method of Lyapunov***

Example: Investigate on stability by the second (direct) method of Lyapunov the linearized (linear) ACS which description is set in the state-space:

$$\begin{cases} \dot{x}_1 = -6x_1 + 2x_2 \\ \dot{x}_2 = -5x_1 + 3x_2 \end{cases}.$$

Lyapunov's function is given as follows:

$$V(x) = \frac{1}{2}(x_1^2 + x_2^2).$$

You should give geometrical interpretation.

Algorithm and solution

1. We will be convinced that the given function *V(x)* is one of fixed positive-sign, i.e.

$$\begin{cases} V(x) > 0 \; if \;\; x \neq 0 \\ V(x) = 0 \; if \;\; x = 0 \end{cases}.$$

2. It is necessary to define *the sign of its full derivative*:

$$\frac{dV}{dt} = \frac{\partial V}{\partial x_1}\frac{dx_1}{dt} + \frac{\partial V}{\partial x_2}\frac{dx_2}{dt};$$

$$\frac{dV}{dt} = x_1 \; x'_1 + x_2 \; x'_2 .$$

3. We substitute the given system dynamic equations:

$$\frac{dV}{dt} = x_1 \dot{x}_1 + x_2 \dot{x}_2 = x_1(-6x_1 + 2x_2) + x_2(-5x_1 + 3x_2) =$$
$$= -6x_1^2 + 2x_1x_2 - 5x_1x_2 + 3x_2^2;$$

4. It is necessary for asymptotic stability of a system that the full derivative of

$$\frac{dV}{dt} = -6x_1^2 + 2x_1x_2 - 5x_1x_2 + 3x_2^2 < 0;$$

$$-6x_1^2 \neq 0 \Rightarrow \frac{-6x_1^2}{-6x_1^2} - \frac{3x_1x_2}{-6x_1^2} + \frac{3x_2^2}{-6x_1^2} < 0.$$

5. We will designate $\frac{x_2}{x_1} = z$; let's rewrite inequality in new designations and we will solve it:

$$-\frac{1}{2}z^2 + \frac{1}{2}z + 1 < 0; \quad (\textit{multiply (-2)})$$

$$z^2 - z - 2 = 0;$$

$$z_{1,2} = \frac{1}{2} \pm \sqrt{\frac{1}{4} + 2};$$

$$z_{1,2} = \frac{1}{2} \pm \frac{3}{2}; \quad z_1 = -1, z_2 = 2.$$

6. Hence, the system asymptotically is steady across Lyapunov when choosing values of variables in the received range $x_2 \in (-x_1, 2x_1)$.

Conclusion: the following condition asymptotically of the steady movement of the researched dynamic system is found the second method of Lyapunov $x_2 \in (-x_1, 2x_1)$.

Geometrical interpretation:

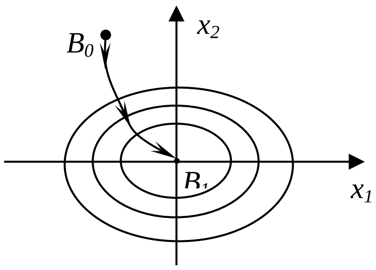

Fig. – The movement of system is asymptotically steady

Task Investigate on stability by the second (direct) method of Lyapunov the linearized (linear) ACS which description is set in the state-space (*on variants*); Lyapunov's function is given as follows:

$$V(x) = 1/2\,(x_1^2 + x_2^2).$$

And you should give geometrical interpretation.

Variants:

1)

$$\begin{cases} x'_1 = -x_1 + 3x_2 \\ x'_2 = 2x_1 - x_2 \end{cases}$$

2)

$$\begin{cases} x'_1 = -5x_1 - 3x_2 \\ x'_2 = 4x_1 + 7x_2 \end{cases}$$

3)

$$\begin{cases} x'_1 = -2x_1 + 2x_2 \\ x'_2 = -7x_1 + 3x_2 \end{cases}$$

4)

$$\begin{cases} x'_1 = 3x_1 - 7x_2 \\ x'_2 = 5x_1 - 2x_2 \end{cases}$$

5)

$$\begin{cases} x'_1 = -2x_1 + 2x_2 \\ x'_2 = -7x_1 + 3x_2 \end{cases}$$

6)

$$\begin{cases} x'_1 = 3x_1 - 3x_2 \\ x'_2 = x_1 - x_2 \end{cases}$$

7)

$$\begin{cases} x'_1 = 5x_1 + 3x_2 \\ x'_2 = -x_1 - 3x_2 \end{cases}$$

8)

$$\begin{cases} x'_1 = -8x_1 + 4x_2 \\ x'_2 = 2x_1 + 5x_2 \end{cases}$$

9)

$$\begin{cases} x'_1 = -6x_1 - x_2 \\ x'_2 = -x_1 + 4x_2 \end{cases}$$

10)

$$\begin{cases} x'_1 = -6x_1 - 9x_2 \\ x'_2 = -5x_1 - 8x_2 \end{cases}$$

11)

$$\begin{cases} x'_1 = 5x_1 - 6x_2 \\ x'_2 = -4x_1 - 8x_2 \end{cases}$$

12)

$$\begin{cases} x'_1 = 2x_1 - 7x_2 \\ x'_2 = 4x_1 - 9x_2 \end{cases}$$